1. A bag contains a large number of coins. It contains only 1p and 2p coins in the ratio 1:3

(a) Find the mean μ and the variance σ^2 of the values of this population of coins.

(3)

A random sample of size 3 is taken from the bag.

(b) List all the possible samples.

(2)

(c) Find the sampling distribution of the mean value of the samples.

(6)
(Total 11 marks)

2. A bag contains a large number of coins. Half of them are 1p coins, one third are 2p coins and the remainder are 5p coins.

(a) Find the mean and variance of the value of the coins.

(4)

A random sample of 2 coins is chosen from the bag.

(b) List all the possible samples that can be drawn.

(3)

(c) Find the sampling distribution of the mean value of these samples.

(6)
(Total 13 marks)

1. (a)

x	1p	2p
$\mathrm{P}(X = x)$	$\frac{1}{4}$	$\frac{3}{4}$

$\mu = 1\times\frac{1}{4} + 2\times\frac{3}{4} = \frac{7}{4}$ or $1\frac{3}{4}$ or 1.75 B1

$\sigma^2 = 1^2\times\frac{1}{4} + 2^2\times\frac{3}{4} - \left(\frac{7}{4}\right)^2$ M1

$= \frac{3}{16}$ or 0.1875 A1 3

Note

B1 1.75 oe

M1 for using $\sum\left(x^2 p\right) - \mu^2$

A1 0.1875 oe

(b) (1,1,1), (1,1,2) any order, (1,2,2) any order, (2,2,2) B1

(1,2,1) (2,1,1) (2,1,2) (2,2,1) all 8 cases considered. B1 2

May be implied by 3 * (1,1,2) and 3 * (1,2,2)

Note

ignore repeats

(c)

$\bar{x}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$\mathrm{P}(\bar{X} = \bar{x})$	$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$	$3\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{64}$	$3\times\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$	$\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$

B1 M1 A1 M1 A1A1 6

Note

1st B1 4 correct means (allow repeats)

1st M1 for p^3 for either of the ends

1st A1 for 1/64or awrt 0.016 **and** 27/64 or awrt 0.422

2nd M1 $3 \times p^2(1 - p)$for either of the middle two $0 < p < 1$

May be awarded for finding the probability of the 3 samples with mean of either 4/3 or 5/3 .

2nd A1 for 9/64 (or 3/64 three times) and 27/64 (or 9/64 three times) accept awrt 3dp.

3rd A1 fully correct table, accept awrt 3dp.

[11]

2. (a)

X	1	2	5
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Mean = $1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 5 \times \frac{1}{6} = 2$ or 0.02 $\Sigma x.p(x)$ need ½ and ⅓ M1 A1

Variance = $1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{3} + 5^2 \times \frac{1}{6} - 2^2 = 2$ or 0.0002 M A1 4

(b) $\Sigma x^2 .p(x) - \lambda^2$

(1,1)
(1,2) and (2,1) B2
(1,5) and (5,1) LHS -1 B1 3

e.e.

(2,2)
(2,5) and (5,2) repeat of “theirs” on RHS B1
(5,5)

(c)

$\bar{x}$	1	1.5	2	3	3.5	5
P($\bar{X}$ = $\bar{x}$)	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$	$\frac{1}{6}$	$2\times\frac{1}{3}\times\frac{1}{6}=\frac{1}{9}$	$\frac{1}{36}$

¼ M1A1

1.5+,-1ee M1 M2 6

[13]

1. A high proportion of candidates attempted the first two parts of this question successfully, with the majority of candidates getting at least one mark for part (b). Those less successful in part (a) either misread the question and ended up with a denominator of 3 for the probabilities or confused formulae for calculating the mean and variance and used, for example, $\sum \frac{xp(x)}{n}$ for the mean or used $E(X^2)$ for σ^2. The solution to part (c) proved beyond the capability of a minority of candidates but, for the majority, many exemplary answers were evident, reflecting sound preparation on this topic. Candidates who found all 8 cases in (b) usually gained four marks in part (c) for calculating the probabilities. For a small percentage of those candidates, calculating the means was difficult and hence completing the table correctly was not possible. A few candidates tried unsuccessfully to use the binomial to answer part (c).

2. In part (a) many candidates were able to calculate the mean accurately, although some divided by random constants. Few drew up a table and many were unable to cope with the 5p coins. The most common error in calculating the variance was the failure to subtract $E(X)^2$.
Most candidates correctly identified 6 possible samples but some failed to realise that combinations such as (1,5) and (5,1) were different and so missed the other 3 possibilities. Only a minority of candidates were able to attempt part (c) of the question with any success, with many candidates having no idea what was meant by 'the sampling distribution of the mean value of the samples'. Most did not find the mean values and if they did, then they were unable to find the probabilities (ninths were common).Very few candidates achieved full marks.